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High-Precision Values of the Gamma Function and of Some Related Coefficients

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Abstract. In this paper we determine numerical values to 80D of the coefficients in the Taylor series expansion $\Gamma^m(s + x) = \Sigma_0^\infty g_k(m, s)x^k$ for certain values of m and s and use these values to calculate $\Gamma(p/q)$ ($p, q = 1, 2, \ldots, 10; p < q$) and $\min_{x>0}\Gamma(x)$ to 80D. Finally, we obtain a high-precision value of the integral $\int_0^\infty(\Gamma(x))^{-1}\,dx$.

1. Introduction. This paper traces its origin from a wish to determine high-precision values of the integrals $F = \int_0^\infty(\Gamma(x))^{-1}\,dx$ and $\int_n^{n+1}(\Gamma(x))^{-1}\,dx$ because the distribution defined by $G(x) = F^{-1}\int_0^x(\Gamma(t))^{-1}\,dt$ may be useful in reliability theory (Fransén [5]). That can also be approximated by a weighted sum of Gamma Distributions or be seen as a special case of using the Fox H-function as a distribution (Carter and Springer [3]). The integrals in question had not been properly studied (to our great astonishment) before. They were not even mentioned in Nielsen's book on the Gamma function [11]. The closest we came in our literature study was to a paper from 1883 by Bourguet [2]. When establishing high-precision values of the integrals Fransén [4] had to calculate the Riemann Zeta function to 80D for integer values, thereby using formulas of Katayama and Ramanujan [8]. These values might be used for many purposes: to determine the coefficients in the Taylor series expansion of $\Gamma^m(s + x)$ for certain interesting values of m and s, the value of $\min_{x>0}\Gamma(x)$, and many other relevant coefficients.

When carrying out the necessary multiple-precision calculation on our DEC-10 computer we have used a Simula Class HIGHPREC developed by a student, Demetre Betsis, at the University of Stockholm.

2. Numerical Values to 80D of the Coefficients in the Taylor Series Expansion of $\Gamma^m(s + x)$ for Certain Values of m and s.

a. *Basic Formulas.* Let m and s be real numbers, $s > 0$. Consider the Taylor series expansion

$$\Gamma^m(s + x) = \sum_{k=0}^{\infty} g_k(m, s)x^k, \tag{2.1}$$

where

$$g_k(m, s) = \frac{1}{k!}\frac{d^k\Gamma^m(s)}{ds^k}.$$

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The coefficients $g_k(m, s)$ may be obtained recursively if we apply Leibniz' differentiation formula to the identity

$$\frac{d\Gamma^m(s)}{ds} = m\psi(s)\Gamma^m(s), \tag{2.2}$$

where $\psi(s)$ is the Psi function $(= \dot{\Gamma}(s)/\Gamma(s))$. We state the main result in the following

THEOREM. *Let m and s be real numbers, $s > 0$. The coefficients $g_k(m, s)$ in the Taylor series expansion*

$$\Gamma^m(s + x) = \sum_{k=0}^{\infty} g_k(m, s)x^k$$

are obtained from the recursion formula

$$(n + 1)g_{n+1}(m, s) = m \sum_{k=0}^{n} (-1)^{k-1} g_{n-k}(m, s)h_k(s),$$

where $h_k(s)$ $(k = 1, 2, \ldots)$ is the Hurwitz Zeta function

$$h_k(s) = \sum_{n=0}^{\infty} \frac{1}{(s + n)^{k+1}}$$

and

$$h_0(s) = -\psi(s) = \lim_{n=\infty} \left(\sum_{k=0}^{n} \frac{1}{s + k} - \ln(n) \right),$$

with the initial value $g_0(m, s) = \Gamma^m(s)$.

A similar result can partly be found in Nielsen's book [11]. We now apply the theorem to some special cases.

b. *Special Cases.* Choosing $s = 1$, we get $g_0(m, 1) = 1$ and $h_0(1) = \gamma$, the Euler constant. Further, we then have $h_k(1) = \zeta(k + 1)$, the Riemann Zeta function, for $k = 1, 2, \ldots$. The computation of values of $\zeta(k)$ for even values of k is straightforward, while we use the Ramanujan formula as described in [8] to compute them in the odd case. In Table I we present these values to 80D for k up to 51.

Putting $m = -1$ and denoting $a_{k+1} = g_k(-1, 1)$, we compute the coefficients using the recursion formula. In Table II we present the values of a_k obtained to 80D for k up to 52. Note that a_k approaches zero very fast. We prove that $\lim_{k=\infty} a_k = 0$ below. We have not been able to derive an approximate expression of a_k or to explain the rather irregular occurrences of plus and minus signs. Already for moderate sizes of s, however, the expansion is properly alternating. Similarly when choosing $m = 1$ and denoting $b_{k+1} = g_k(1, 1)$ we get the values of b_k presented in Table III. We prove also that $\lim_{k=\infty} g_{2k}(1, 1) = 1$ and $\lim_{k=\infty} g_{2k+1}(1, 1) = -1$ below.

Finally, choosing $s = 3/2$, we get $-h_0(s) = \psi(s) = 2 - \gamma - 2 \ln 2$ and $h_k(s) = (2^{k+1} - 1)\zeta(k + 1) - 2^{k+1}$ (see [1]) for $k = 1, 2, \ldots$. We denote for $m = 1$, $c_{k+1} = g_k(1, 3/2)$ and for $m = -1$, $d_{k+1} = g_k(-1, 3/2)$, and present the values of c_k and d_k in Tables IV and V. Note that $g_k(1, 3/2) \approx (-1)^k(2/3)^{k+1}$ and $\lim_{k=\infty} g_k(-1, 3/2) = 0$.

Some of the results presented in Tables I–V have partly been published previously. A paper by J. W. Wrench [13] gives the coefficients a_k to 31D for $k = 2(1)41$. Last-figure corrections appeared in *Math. Comp.*, v. 27, 1973, pp. 681–682, MTE 505. A. H. Morris has compiled two unpublished tables, deposited in the UMT file. The first one, [9], gives $\zeta(k)$ to 70D for $k = 2(1)90$. The second one, [10], includes a tabulation of a_k to 70D for $k = 1(1)73$.

c. *A Draft Proof.* In the text (Section 2b) it is remarked that $\lim_{k=\infty} a_k = 0$ and that $\lim_{k=\infty} g_{2k}(1, 1) = -\lim_{k=\infty} g_{2k+1}(1, 1) = 1$.

By using ordinary residue calculus one sees that

$$\frac{1}{2\pi i} \int z^{-k} \frac{1}{\Gamma(z)} dz = a_{k-1}, \tag{2.3}$$

where the integration is carried out around $|z| = 1$. We put $z = e^{i\theta}$. Then

$$a_k = \frac{1}{2\pi} \int_0^{2\pi} (\cos(k\theta) - i \sin(k\theta)) \frac{1}{\Gamma(e^{i\theta})} d\theta. \tag{2.4}$$

Using a well-known lemma of Riemann-Lebesgue, one gets $\lim_{k=\infty} a_k = 0$. To prove the other results one starts with the identity

$$\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx = R_1(s) + R_2(s), \tag{2.5}$$

where

$$R_1(s) = \int_0^1 x^{s-1} e^{-x} dx = \sum_{k=0}^{\infty} \frac{(-1)^k}{(s+k)k!}$$

and

$$R_2(s) = \int_1^\infty x^{s-1} e^{-x} dx.$$

Differentiating Eq. (2.5) n times with respect to s, we get

$$\Gamma^{(n)}(s) \frac{1}{n!} = \sum_{k=0}^{\infty} \frac{(-1)^{k+n}}{(s+k)^{n+1} k!} + \frac{R_2^{(n)}(s)}{n!}. \tag{2.6}$$

When $s = 1$ and $n \longrightarrow \infty$ "everything" in the right-hand side of Eq. (2.6) approaches zero except the term $(-1)^n$. Q.E.D.

3. Numerical Values to 80D of $\Gamma(p/q)$; $p, q = 1, 2, \ldots, 10$, $p < q$.

a. *By Taylor Series Expansions.* We shall calculate in all 32 different values $\Gamma(p/q)$, where $p/q \in I$ and $I = I_1 \cup I_2 \cup I_3 \cup I_4 \cup I_5$, where

$$I_1 = \left\{\frac{1}{10}, \frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}\right\},$$

$$I_2 = \left\{\frac{1}{5}, \frac{2}{9}, \frac{1}{4}, \frac{2}{7}, \frac{3}{10}\right\},$$

$$I_3 = \left\{\frac{1}{3}, \frac{3}{8}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{1}{2}, \frac{5}{9}, \frac{4}{7}, \frac{3}{5}, \frac{5}{8}, \frac{2}{3}\right\},$$

$$I_4 = \left\{\frac{7}{10}, \frac{5}{7}, \frac{3}{4}, \frac{7}{9}, \frac{4}{5}\right\},$$

$$I_5 = \left\{\frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{1}{1}\right\}.$$

We use the Taylor series expansions we have, viz.

$$f(x) = \frac{1}{\Gamma(1+x)} = \sum_{k=0}^{\infty} g_k(-1, 1)x^k \tag{3.1}$$

and

$$g(x) = \frac{1}{\Gamma(3/2+x)} = \sum_{k=0}^{\infty} g_k\left(-1, \frac{3}{2}\right)x^k. \tag{3.2}$$

We have to calculate $xf(x)$ and $(x + ½)g(x)$ with a precision a little greater than $0.5 \cdot 10^{-80}$, whereupon we invert. Therefore, we must have

$$|g_{51}(-1, 1)|x^{52} \lesssim 0.5 \cdot 10^{-80}, \qquad |½g_{51}(-1, 3/2)||x|^{51} \lesssim 0.5 \cdot 10^{-80}.$$

The corresponding values of x are $0 < x < 0.1970$ and $|x| < 0.1964$. If we also allow negative values of x in $f(x)$, we get $-0.1908 < x < 0$.

We can hereby calculate the following values:

(3.3) with $xf(x)$ $\Gamma\left(\frac{p}{q}\right)$ where $\frac{p}{q} \in I_1: x = \frac{p}{q}$ in Eq. (3.1),

(3.4) with $(x + ½)g(x)$ $\Gamma\left(\frac{p}{q}\right)$ where $\frac{p}{q} \in I_3: x + ½ = \frac{p}{q}$ in Eq. (3.2),

(3.5) with $f(x)$ $\Gamma\left(\frac{p}{q}\right)$ where $\frac{p}{q} \in I_5: 1 + x = \frac{p}{q}$ in Eq. (3.1).

The values $\Gamma(p/q)$ where $p/q \in I_2 \cup I_4$ cannot be calculated in this simple way. For these missing values we use the duplication formula

$$\Gamma(2x)\sqrt{\pi} = 2^{2x-1}\Gamma(x)\Gamma(x + ½). \tag{3.6}$$

The 32 values of $\Gamma(p/q)$ appear in Table VII.

b. *By Elliptic Integrals*. As is shown by Wrigge [14]–[16] and Glasser and Wood [7] and many others, there is a close relationship between certain values of the gamma function and the complete elliptic integral $K(t)$. The easiest way of getting a good value of $K(t)$ is to use the arithmetic-geometric mean $M(t)$ (see [1]), which will give an accurate value of $K(t)$ to at least 80D in less than 10 steps. We have

$$K(t) = \frac{\pi}{2M(t)}. \tag{3.7}$$

It is, e.g., known that (Fransén [4])

$$\Gamma^2\left(\frac{1}{4}\right) = 4\sqrt{\pi}K\left(\frac{1}{\sqrt{2}}\right), \tag{3.8}$$

$$\Gamma^2\left(\frac{1}{8}\right) = 16K(\sqrt{2}-1)2^{-3/4}\Gamma\left(\frac{1}{4}\right). \tag{3.9}$$

By using the duplication and reflection formulas for the Gamma function (see [1]) it is possible to get easy-to-calculate expressions for all $\Gamma(p/8)$, $p = 1, 2, \ldots, 8$. Such values are presented to 80D in [4]. Similar results hold for $\Gamma(p/6)$ (see [7]). The values thus calculated agree excellently with the values calculated by the Taylor series method in a. or by other methods as presented in [6].

4. Numerical Values to 80D of x_0 and $\Gamma(x_0) = \min_{x>0}\Gamma(x)$. Instead of $\min_{x>0}\Gamma(x)$ we study $\max_{x>0}(\Gamma(x))^{-1}$ and, thereby, use the Taylor series expansion

$$g(x) = \frac{1}{\Gamma(3/2 + x)} = \sum_{k=0}^{\infty} g_k\left(-1, \frac{3}{2}\right)x^k. \tag{4.1}$$

By making a more and more refined tabulation of $g(x)$ we managed to get a value of x_0 to 37D and of $\min \Gamma(x)$ to 74D. In order to get a better value of x_0 we studied

$$\dot{g}(x) = \sum_{k=1}^{\infty} kg_k\left(-1, \frac{3}{2}\right)x^{k-1}, \tag{4.2}$$

and then made use of "repeated" inverse Bessel interpolation. We thus got

$$\begin{aligned} x_0 = 1.\ & 46163\ 21449\ 68362\ 34126 \\ & 26595\ 42325\ 72132\ 84681 \\ & 96204\ 00644\ 63512\ 95988 \\ & 40859\ 87864\ 40353\ 80181 \end{aligned} \tag{4.3}$$

and

$$\begin{aligned} \Gamma(x_0) = 0.\ & 88560\ 31944\ 10888\ 70027 \\ & 88159\ 00582\ 58873\ 32079 \\ & 51533\ 66990\ 34488\ 71200 \\ & 16587\ 51362\ 27417\ 39635. \end{aligned} \tag{4.4}$$

5. Numerical Values to 60D of $\int_n^{n+1}(\Gamma(x))^{-1}\,dx$ ($n = -10, -9, \ldots, 48$) and $\int_0^\infty(\Gamma(x))^{-1}\,dx$. The original problem for this paper was to determine a high-precision value of the integral $F = \int_0^\infty(\Gamma(x))^{-1}\,dx$ (named Fransén's constant in H. P. Robinson's extensive file of mathematical constants). In [4] Fransén did that using the Euler-Maclaurin formula and got the value of F to 65D. Here we will use the Taylor series methods in Section 2 to calculate $\int_n^{n+1}(\Gamma(x))^{-1}\,dx$ for $n \geqslant 0$.

Using routine manipulation, we get

$$\begin{aligned} \int_n^{n+1}(\Gamma(x))^{-1}\,dx = & \int_0^{1/4}(\Gamma(x+n))^{-1}\,dx \\ & + \int_{-1/4}^{+1/4}(\Gamma(x+n+½))^{-1}\,dx \\ & + \int_{-1/4}^{0}(\Gamma(x+n+1))^{-1}\,dx. \end{aligned} \tag{5.1}$$

But

$$\int_v^u (\Gamma(s+x))^{-1}\,dx = \sum_{k=0}^{\infty} g_k(-1, s)\,\frac{u^{k+1}-v^{k+1}}{k+1}. \tag{5.2}$$

Using the methods in Section 2 and/or some simpler recurrence relations, the coefficients $g_k(-1, n)$ and $g_k(-1, 3/2 + n)$ were calculated for the necessary values of n and the integration carried out. The results to 60D appear in Table VI.

To calculate $\int_n^{n+1}(\Gamma(x))^{-1}\,dx$ for $n < 0$ we used Eq. (5.1) valid also for $n < 0$. Furthermore,

$$\frac{1}{\Gamma(-n+x)} = x(x-1)(x-2)\ldots(x-n)\,\frac{1}{\Gamma(1+x)} \tag{5.3}$$

and

$$\frac{1}{\Gamma(-n+\tfrac12+x)} = (\tfrac12+x)(-\tfrac12+x)\ldots(-n+\tfrac12+x)\,\frac{1}{\Gamma\left(\frac{3}{2}+x\right)}. \tag{5.4}$$

Using the Eqs. (5.3), (5.4) and the Taylor series expansions of $(\Gamma(1+x))^{-1}$ and $(\Gamma(3/2+x))^{-1}$ we calculated the integrals also for negative values of n. The results to 60D are presented in Table VI.

6. Tables. In this section we present the tables previously mentioned, i.e.,

I. The values of γ, the Euler constant and $\zeta(k)$, $k = 2, 3, \ldots, 51$, the Riemann Zeta function, to 80D.

II. The coefficients $g_k(-1, 1)$ in the Taylor series expansion $(\Gamma(1+x))^{-1} = \Sigma_0^\infty g_k(-1, 1)x^k$ to 80D. Note that $a_{k+1} = g_k(-1, 1)$.

III. The coefficients $g_k(1, 1)$ in the Taylor series expansion $\Gamma(1+x) = \Sigma_0^\infty g_k(1, 1)x^k$ to 80D. Note that $b_{k+1} = g_k(1, 1)$.

IV. The coefficients $g_k(1, 3/2)$ in the Taylor series expansion $\Gamma(3/2+x) = \Sigma_0^\infty g_k(1, 3/2)x^k$ to 80D. Note that $c_{k+1} = g_k(1, 3/2)$.

V. The coefficients $g_k(-1, 3/2)$ in the Taylor series expansion $(\Gamma(3/2+x))^{-1} = \Sigma_0^\infty g_k(-1, 3/2)x^k$ to 80D. Note that $d_{k+1} = g_k(-1, 3/2)$.

VI. The values of $\int_0^\infty(\Gamma(x))^{-1}\,dx$ and $\int_n^{n+1}(\Gamma(x))^{-1}\,dx$ ($n = -10, -9, \ldots, 48$) to 60D.

VII. The values of $\Gamma(p/q)$, $p, q = 1, 2, \ldots, 10$: $p < q$, to 80D.

Tabulated values are commonly rounded with a last-figure error not exceeding half a unit.

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TABLE I

Values of the Euler constant, Gamma, and the Riemann Zeta function for integral values.

Gamma	= 0.57721 56649 01532 86060 65120 90082 40243 10421 59335 93992 35988 05767 23488 48677 26777 66467
Zeta(2)	= 1.64493 40668 48226 43647 24151 66646 02518 92189 49901 20679 84377 35558 22937 00074 70403 20087
Zeta(3)	= 1.20205 69031 59594 28539 97381 61511 44999 07649 86292 34049 88817 92271 55534 18382 05786 31309
Zeta(4)	= 1.08232 32337 11138 19151 60036 96541 16790 27747 50951 91872 69076 82976 21544 41206 16186 96885
Zeta(5)	= 1.03692 77551 43369 92633 13654 86457 03416 80570 80919 50191 28119 74192 67790 38035 89786 28148
Zeta(6)	= 1.01734 30619 84449 13971 45179 29790 92052 79018 17490 03285 35618 42408 66400 43321 82901 95790
Zeta(7)	= 1.00834 92773 81922 82683 97975 49849 79675 95998 63560 56523 87064 17283 13657 16014 78317 35574
Zeta(8)	= 1.00407 73561 97944 33937 86852 38508 65246 52589 60790 64985 00203 29110 20265 25829 52574 74881
Zeta(9)	= 1.00200 83928 26082 21441 78527 69232 41206 04856 05851 39488 87565 48596 61590 97850 53390 25840
Zeta(10)	= 1.00099 45751 27818 08533 71459 58900 31901 70060 19531 56447 75172 57788 99463 62914 65151 91295
Zeta(11)	= 1.00049 41886 04119 46455 87022 82526 46993 64686 06435 75820 86171 19141 43610 00540 59798 21981
Zeta(12)	= 1.00024 60865 53308 04829 86379 98047 73967 09604 16088 45800 34045 33040 95213 32520 19681 94091
Zeta(13)	= 1.00012 27133 47578 48914 67518 36526 35739 57142 75105 89550 98451 36702 67162 08967 26829 84421
Zeta(14)	= 1.00006 12481 35058 70482 92585 45105 13533 37474 81696 16915 45494 82755 20225 28629 41023 17742
Zeta(15)	= 1.00003 05882 36307 02049 35517 28510 64506 25876 27948 70685 81775 06569 93289 33322 67156 34228
Zeta(16)	= 1.00001 52822 59408 65187 17325 71487 63672 20232 37388 99047 15311 53105 20358 87870 87027 95315
Zeta(17)	= 1.00000 76371 97637 89976 22736 00293 56302 92130 88249 09026 26790 95379 84397 29356 43290 28246
Zeta(18)	= 1.00000 38172 93264 99983 98564 61644 62193 97304 54697 21895 33311 43174 42998 76300 39542 65005
Zeta(19)	= 1.00000 19082 12716 55393 89256 56957 79510 13532 58571 14483 86302 35933 04676 18239 49705 34131
Zeta(20)	= 1.00000 09539 62033 87279 61131 52038 68344 93459 43794 18741 05957 50056 48985 11375 13731 14390
Zeta(21)	= 1.00000 04769 32986 78780 64631 16719 60437 30459 66446 69478 49376 00207 48737 65968 39087 89816
Zeta(22)	= 1.00000 02384 50502 72773 29900 03648 18675 29949 35041 82177 96582 69849 60311 64744 58935 62291
Zeta(23)	= 1.00000 01192 19925 96531 10730 67788 71888 23263 87254 99778 45198 58603 22579 72362 43730 42744
Zeta(24)	= 1.00000 00596 08189 05125 94796 12440 20793 58012 27503 91883 73027 95864 24697 23217 24495 35547
Zeta(25)	= 1.00000 00298 03503 51465 22801 86063 70506 93660 11844 73091 95433 12398 68133 90133 84460 76746
Zeta(26)	= 1.00000 00149 01554 82836 50412 34658 50663 06986 28864 78816 78859 10547 43596 87899 71296 74486
Zeta(27)	= 1.00000 00074 50711 78983 54294 91981 00417 06041 19454 71903 18825 65829 99323 95783 52147 60627
Zeta(28)	= 1.00000 00037 25334 02478 84570 54819 20401 84024 23232 89305 92958 11519 76933 47061 69604 96030
Zeta(29)	= 1.00000 00018 62659 72351 30490 06403 90994 54169 48061 66533 04692 00665 77489 38055 58091 69327
Zeta(30)	= 1.00000 00009 31327 43241 96681 82871 76473 50212 19813 56795 51368 16185 00861 33604 41960 67294
Zeta(31)	= 1.00000 00004 65662 90650 33784 07298 92332 51220 07106 26918 53369 47307 37297 16933 71175 66989
Zeta(32)	= 1.00000 00002 32831 18336 76505 49200 14559 75940 49502 48298 22845 30311 07760 22583 87912 18939
Zeta(33)	= 1.00000 00001 16415 50172 70051 97759 29738 35456 30951 65224 71727 63593 25651 77399 47029 12462
Zeta(34)	= 1.00000 00000 58207 72087 90270 08892 43685 98910 63054 17312 26046 17215 95507 16881 24163 07140
Zeta(35)	= 1.00000 00000 29103 85044 49709 96869 29425 22788 40464 10698 19874 33032 25621 02548 25640 48890
Zeta(36)	= 1.00000 00000 14551 92189 10419 84235 92963 22453 18420 98380 88941 24038 06913 95422 18571 74587
Zeta(37)	= 1.00000 00000 07275 95983 50574 81014 52086 90123 38059 26485 09255 55466 10770 57969 42638 43837
Zeta(38)	= 1.00000 00000 03637 97954 73786 51190 23723 63558 73273 51264 60283 84897 46994 79515 94042 71425
Zeta(39)	= 1.00000 00000 01818 98965 03070 65947 58483 21007 30085 03058 93096 18664 07053 52512 53356 50932
Zeta(40)	= 1.00000 00000 00909 49478 40263 88928 25331 18386 94908 75386 00009 90878 82850 54797 10112 02537
Zeta(41)	= 1.00000 00000 00454 74737 83042 15402 67991 12029 48857 03390 45299 11438 62808 12340 35905 00260
Zeta(42)	= 1.00000 00000 00227 37368 45824 65251 52268 21577 97869 12138 29821 98915 87258 05336 47882 22296
Zeta(43)	= 1.00000 00000 00113 68684 07680 22784 93491 04838 02590 64374 35902 84251 79989 90122 76309 35911
Zeta(44)	= 1.00000 00000 00056 84341 98762 75856 09277 18296 75240 68553 05715 88993 88351 68064 44440 46359
Zeta(45)	= 1.00000 00000 00028 42170 97688 93018 55455 07370 49426 62074 36882 65309 83382 76290 62781 75669
Zeta(46)	= 1.00000 00000 00014 21085 48280 31606 76983 43071 41739 53767 86986 05633 95195 74517 00244 20901
Zeta(47)	= 1.00000 00000 00007 10542 73952 10852 71287 73544 79956 80002 27420 43593 68768 83638 28887 75095
Zeta(48)	= 1.00000 00000 00003 55271 36913 37113 67329 84695 34059 34299 21456 55503 06261 50125 17934 04758
Zeta(49)	= 1.00000 00000 00001 77635 68435 79120 32747 33490 14400 27957 01555 08575 32695 19787 59241 23327
Zeta(50)	= 1.00000 00000 00000 88817 84210 93081 59030 96091 38639 13863 25608 87146 46446 66449 76989 90084
Zeta(51)	= 1.00000 00000 00000 44408 92103 14381 33641 97770 94026 81213 36459 60307 02441 80285 97831 15341

TABLE II

Values of the coefficients in the expansion of the inverted Gamma function for s = 1.

a(1) =	1.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000
a(2) =	0.57721 56649 01532 86060 65120 90082 40243 10421 59335 93992 35988 05767 23488 48677 26777 66467
a(3) =	-0.65587 80715 20253 88107 70195 15145 39048 12797 66380 47858 43472 92362 44568 38708 38353 72210
a(4) =	-0.04200 26350 34095 23552 90039 34875 42981 87113 94500 40110 60935 22065 81297 61800 96875 97599
a(5) =	0.16653 86113 82291 48950 17007 95102 10523 57177 81502 24717 43405 70468 90317 89938 66056 47425
a(6) =	-0.04219 77345 55544 33674 82083 01289 18739 13016 52684 18982 24863 76918 87327 54590 11185 58900
a(7) =	-0.00962 19715 27876 97356 21149 21672 34819 89753 62942 25211 30021 05138 86262 73116 73514 46074
a(8) =	0.00721 89432 46663 09954 23950 10340 44657 27099 04800 88023 83180 01094 78117 36225 94974 15854
a(9) =	-0.00116 51675 91859 06511 21139 71084 01838 86668 09333 79538 40574 43407 50527 56200 25848 16653
a(10) =	-0.00021 52416 74114 95097 28157 29963 05364 78064 78241 92337 83387 50350 26748 90856 39463 71678
a(11) =	0.00012 80502 82388 11618 61531 98626 32816 43233 94892 09969 36772 14900 54583 80412 03552 04347
a(12) =	-0.00002 01348 54780 78823 86556 89391 42102 18183 82294 83329 79791 15261 16267 09082 29186 18897
a(13) =	-0.00000 12504 93482 14267 06573 45359 47383 30922 42322 65562 11539 59815 34992 31574 91212 45561
a(14) =	0.00000 11330 27231 98169 58823 74129 62033 07449 43324 00483 86210 75654 29550 53954 60408 42730
a(15) =	-0.00000 02056 33841 69776 07103 45015 41300 20572 83651 25790 26293 37945 34683 17253 32456 80371
a(16) =	0.00000 00061 16095 10448 14158 17862 49868 28553 42867 27586 57197 12320 86732 40292 77235 07435
a(17) =	0.00000 00050 02007 64446 92229 30055 66504 80599 91303 04461 27424 94481 71895 33788 77374 72132
a(18) =	-0.00000 00011 81274 57048 70201 44588 12656 54365 05577 73875 95049 32587 59096 18926 31696 43391
a(19) =	0.00000 00001 04342 67116 91100 51049 15403 32312 25019 14007 09823 12581 21210 87107 39273 47588
a(20) =	0.00000 00000 07782 26343 99050 71254 04993 73113 60777 22606 80861 81392 93881 94355 07326 92987
a(21) =	-0.00000 00000 03696 80561 86422 05708 18781 58780 85766 23657 09634 51360 99513 64845 46554 43000
a(22) =	0.00000 00000 00510 03702 87454 47597 90154 81322 86323 18027 26886 06970 76321 17350 10485 65735
a(23) =	-0.00000 00000 00020 58326 05356 65067 83222 42954 48552 37419 74609 10808 10147 18805 81964 44349
a(24) =	-0.00000 00000 00005 34812 25394 23017 98237 00173 18727 93994 89897 15478 12068 21116 80954 93211
a(25) =	0.00000 00000 00001 22677 86282 38260 79015 88938 46622 42242 81654 55750 45632 13660 11359 99606
a(26) =	-0.00000 00000 00000 11812 59301 69745 87695 13764 58684 22978 31211 55729 18048 47879 83750 81233
a(27) =	0.00000 00000 00000 00118 66922 54751 60033 25797 77242 92867 40710 88494 07966 48271 10740 06109
a(28) =	0.00000 00000 00000 00141 23806 55318 03178 15558 03947 56670 90370 86350 75033 45256 25641 22263
a(29) =	-0.00000 00000 00000 00022 98745 68443 53702 06592 47858 06336 99260 28450 59314 19036 70148 89830
a(30) =	0.00000 00000 00000 00001 71440 63219 27337 43338 39633 70267 25706 68126 56062 51743 31746 49858
a(31) =	0.00000 00000 00000 00000 01337 35173 04936 93114 86478 13951 22268 02287 50594 71761 89478 98583
a(32) =	-0.00000 00000 00000 00000 02054 23355 17666 72789 32502 53513 55733 79668 20379 35238 73641 27301
a(33) =	0.00000 00000 00000 00000 00273 60300 48607 99984 48315 09904 33098 20148 65311 69583 63633 70165
a(34) =	-0.00000 00000 00000 00000 00017 32356 44591 05166 39057 42845 15647 79799 06974 91087 94998 41377
a(35) =	-0.00000 00000 00000 00000 00000 23606 19024 49928 72873 43450 73542 75310 07926 41355 21453 70486
a(36) =	0.00000 00000 00000 00000 00000 18649 82941 71729 44307 18413 16187 86668 98945 86842 90736 68232
a(37) =	-0.00000 00000 00000 00000 00000 02218 09562 42071 97204 39971 69136 26860 37973 17795 00675 67580
a(38) =	0.00000 00000 00000 00000 00000 00129 77819 74947 99366 88244 14486 33059 41656 19499 86463 91332
a(39) =	0.00000 00000 00000 00000 00000 00001 18069 74749 66528 40622 27454 15509 97151 85596 84637 84158
a(40) =	-0.00000 00000 00000 00000 00000 00001 12458 43492 77088 09029 36546 74261 43951 21194 11795 58301
a(41) =	0.00000 00000 00000 00000 00000 00000 12770 85175 14086 62039 90206 67775 11246 47748 77206 56005
a(42) =	-0.00000 00000 00000 00000 00000 00000 00739 14511 69615 14082 34612 89330 10855 28237 10568 99245
a(43) =	0.00000 00000 00000 00000 00000 00000 00001 13475 02575 54215 76095 41652 59469 30639 30086 12196
a(44) =	0.00000 00000 00000 00000 00000 00000 00004 63913 46410 58722 02994 48049 07952 22846 30579 68680
a(45) =	-0.00000 00000 00000 00000 00000 00000 00000 53473 36818 43919 88750 77418 19670 98933 20904 88591
a(46) =	0.00000 00000 00000 00000 00000 00000 00000 03207 99592 36133 52622 86123 72790 82794 39109 01464
a(47) =	-0.00000 00000 00000 00000 00000 00000 00000 00044 45829 73655 07568 82101 59035 21246 43637 40144
a(48) =	-0.00000 00000 00000 00000 00000 00000 00000 00013 11174 51888 19887 12901 05849 43899 22190 23663
a(49) =	0.00000 00000 00000 00000 00000 00000 00000 00001 64703 33525 43813 88681 82593 27906 39414 53996
a(50) =	-0.00000 00000 00000 00000 00000 00000 00000 00000 10562 33178 50358 12186 00561 07153 82850 49997
a(51) =	0.00000 00000 00000 00000 00000 00000 00000 00000 00267 84429 82643 04947 83549 63071 89085 19485
a(52) =	0.00000 00000 00000 00000 00000 00000 00000 00000 00024 24715 49485 17826 89673 03293 83709 21241

TABLE III

Values of the coefficients in the expansion of the Gamma function for $s = 1$.

b(1) =	1.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000
b(2) =	-0.57721 56649 01532 86060 65120 90082 40243 10421 59335 93992 35988 05767 23488 48677 26777 66467
b(3) =	0.98905 59953 27972 55539 53956 51500 63470 79391 83520 72821 40904 43195 78368 61366 32049 47877
b(4) =	-0.90747 90760 80886 28901 65601 67356 27511 49286 11449 07256 37609 41331 15405 04651 82372 23069
b(5) =	0.98172 80868 34400 18733 63802 94021 85085 03605 73679 72346 54154 04957 45559 38568 39248 69345
b(6) =	-0.98199 50689 03145 20210 47014 13791 37467 55174 26507 14719 89304 99967 19048 80063 64964 05004
b(7) =	0.99314 91146 21276 19315 38672 53328 65849 80374 90755 23943 16264 78246 06777 46309 93337 89471
b(8) =	-0.99600 17604 42431 53397 00784 19664 56668 67352 98809 55457 89153 46292 61939 78976 92730 81415
b(9) =	0.99810 56937 83128 92197 85754 03088 36723 75239 68524 79018 34093 17824 15586 78509 37181 57519
b(10) =	-0.99902 52676 21954 86779 46780 59648 88808 85323 03963 52566 09266 60460 49520 89440 53968 52065
b(11) =	0.99951 56560 72777 44106 70508 77594 37019 44345 03297 99459 96618 87795 53794 16479 85013 58241
b(12) =	-0.99975 65975 08601 28702 58424 49140 60923 59969 51385 62883 01160 02987 61907 94492 39829 67513
b(13) =	0.99987 82713 15133 27572 61716 42590 00321 93876 29108 95432 35135 96499 86723 61444 96372 24390
b(14) =	-0.99993 90642 06444 31683 58522 31368 95513 18579 43502 82804 05438 51445 15253 46144 07783 23981
b(15) =	0.99996 95177 63482 10449 86114 05091 95350 72655 28042 47987 55488 65763 48316 84486 59036 21443
b(16) =	-0.99998 47526 99377 04874 37096 31724 44753 83260 83325 77144 87187 73327 14971 24085 59359 28436
b(17) =	0.99999 23744 79073 21585 53950 94505 10782 58338 16344 69466 23437 59939 15975 39564 68407 77736
b(18) =	-0.99999 61865 89473 31202 89649 57795 61431 38020 17312 43262 96264 91003 35457 52459 43096 37567
b(19) =	0.99999 80930 81130 89205 18661 91514 59489 77316 95571 98830 07247 42117 16990 05886 05646 00075
b(20) =	-0.99999 90464 68911 15771 74868 79470 54372 63246 96163 24955 25356 39485 37951 58126 65559 98870
b(21) =	0.99999 95232 10605 73957 52392 92991 06456 81680 99689 08595 02411 15566 89661 93859 34343 77032
b(22) =	-0.99999 97615 97344 38057 09247 01062 58744 74860 97486 06106 79761 15804 89247 54105 43853 93630
b(23) =	0.99999 98807 96019 16841 66504 18404 24924 05265 35466 12259 91412 90364 30956 36225 64234 55058
b(24) =	-0.99999 99403 97124 98374 58628 87976 75081 78480 70342 34988 01115 11147 99845 55286 43596 09524
b(25) =	0.99999 99701 98267 58235 55744 96192 51141 98133 93331 05311 53702 60600 44530 05330 03611 41237
b(26) =	-0.99999 99850 99035 47504 70871 68476 76946 50623 93258 45458 43468 42905 34506 54146 37210 86400
b(27) =	0.99999 99925 49484 96246 47047 99253 72366 70398 87448 08553 98289 22009 20621 80220 82484 90290
b(28) =	-0.99999 99962 74731 55543 69143 33339 28727 57230 56827 55466 89035 41647 03001 13697 93514 95990
b(29) =	0.99999 99981 37362 13559 46670 62812 73498 73662 16664 36388 53269 11266 06099 68930 66551 38883
b(30) =	-0.99999 99990 68679 85370 78792 17910 28525 58993 63865 14797 10538 55556 63652 87611 28711 83105
b(31) =	0.99999 99995 34339 52214 54213 38976 33392 29355 42708 48053 10417 69173 95130 24666 82328 65762
b(32) =	-0.99999 99997 67169 62616 68609 99055 77699 57113 80146 58538 46642 27035 39958 12151 57182 27574
b(33) =	0.99999 99998 83584 76811 40614 12144 33725 11380 04193 65578 38013 30269 38598 69010 10603 32961
b(34) =	-0.99999 99999 41792 36906 70528 49822 71872 51768 40486 60470 81390 48199 76676 58386 67194 70938
b(35) =	0.99999 99999 70896 17953 68200 96608 90584 08312 49885 84544 42256 21830 34948 51881 62591 16490
b(36) =	-0.99999 99999 85448 08810 28295 10703 86516 33114 56842 76628 90964 78557 67215 67939 92009 77029
b(37) =	0.99999 99999 92724 04349 62183 02305 31125 59562 70723 22128 96118 33293 84663 18671 51876 65851
b(38) =	-0.99999 99999 96362 02156 30429 31646 52931 24475 06762 42713 56100 22386 52560 97617 65315 96127
b(39) =	0.99999 99999 98181 01071 98325 42192 88049 41138 18322 77943 53591 93637 62116 85866 09849 78422
b(40) =	-0.99999 99999 99090 50533 93532 50603 07999 67918 15632 65304 67698 65514 79163 38316 60450 38611
b(41) =	0.99999 99999 99545 25266 28222 73649 38304 56804 97889 34445 69833 14009 39994 14834 16264 43744
b(42) =	-0.99999 99999 99772 62632 91263 50068 66751 57690 47623 95962 25072 62801 05408 82448 35143 07778
b(43) =	0.99999 99999 99886 31316 38015 78730 99034 07802 58258 70406 38240 02685 30789 66797 91847 33938
b(44) =	-0.99999 99999 99943 15658 16469 23751 54501 22513 67242 74506 10189 56126 36214 29442 33318 51727
b(45) =	0.99999 99999 99971 57829 07388 39959 57983 22160 15192 01567 57545 05116 67538 83008 72142 77634
b(46) =	-0.99999 99999 99985 78914 53412 12663 17329 62179 85436 85324 88272 58742 76261 92447 85040 86160
b(47) =	0.99999 99999 99992 89457 26612 03889 90966 13410 35809 53299 72637 68872 93563 02268 98016 63603
b(48) =	-0.99999 99999 99996 44728 63274 67797 02797 04483 84165 49645 59679 80929 27883 77811 90235 63128
b(49) =	0.99999 99999 99998 22364 31626 89182 36306 50722 66169 97315 41557 27183 51523 40265 16810 02934
b(50) =	-0.99999 99999 99999 11182 15809 96352 42073 40154 16512 26935 39198 49178 90362 57498 91079 92032
b(51) =	0.99999 99999 99999 55591 07903 82096 61247 78598 29551 61102 78137 65474 02255 93051 45294 16835
b(52) =	-0.99999 99999 99999 77795 53951 52355 10420 34656 87615 95224 82168 26197 72031 72466 04994 22854

TABLE IV

Values of the coefficients in the expansion of the Gamma function for $s = 3/2$.

c(1) =	0.88622 69254 52758 01364 90837 41670 57259 13987 74728 06119 35641 06903 89492 64556 42295 51609
c(2) =	0.03233 83974 48885 01382 88698 84268 97030 77813 34788 87050 70206 36641 01945 98595 99162 17310
c(3) =	0.41481 34536 88301 16823 00376 23111 35634 28489 09963 37042 23679 77719 75186 72661 53692 42118
c(4) =	-0.10729 48045 64772 21168 75419 56389 70966 20545 75923 82129 83009 38639 21109 25010 51470 65111
c(5) =	0.14464 53590 44621 54303 83322 10253 88452 40700 26861 53098 14284 13968 13793 81159 21866 83805
c(6) =	-0.07752 30522 99854 20344 46773 21416 50897 04742 16125 82748 32689 98953 06131 91086 87003 27711
c(7) =	0.05861 03038 17176 28950 41887 37819 14405 71055 54892 49810 41604 63949 88584 17696 43212 84699
c(8) =	-0.03800 19355 54865 13025 20510 71015 03415 52379 66692 62041 88995 98331 43979 82709 03702 22051
c(9) =	0.02583 76064 55756 20389 37000 08736 64624 62969 62568 25229 61249 47665 54531 45610 51421 42953
c(10) =	-0.01722 24431 13464 62506 58306 84260 38043 06974 60083 97127 20406 21622 85099 10670 93449 78632
c(11) =	0.01152 25153 92399 22834 77287 32942 17459 05333 95721 40395 14716 17920 32958 91599 29065 18147
c(12) =	-0.00769 02113 64241 57866 25887 86617 69250 21593 80742 47981 01565 75780 88757 51219 48279 84295
c(13) =	0.00513 16435 01912 38754 09072 03354 32845 98970 30651 46507 49394 36462 31904 81235 66209 27091
c(14) =	-0.00342 28024 97359 70609 69796 85004 86505 43079 80807 07929 91521 72606 30447 00181 83299 26725
c(15) =	0.00228 25897 63790 26741 39310 80530 31818 45352 76813 90405 82695 37026 75690 83498 77869 82913
c(16) =	-0.00152 20100 71124 42832 08129 12968 77465 83302 59316 02021 86016 31910 74780 83475 77592 67488
c(17) =	0.00101 47877 42151 47788 22410 83948 50899 46922 74047 58873 39039 37789 10126 35348 11730 99948
c(18) =	-0.00067 65708 41060 01236 72918 42178 80653 68956 95772 42457 64648 66473 73239 88956 15111 42475
c(19) =	0.00045 10655 25395 65954 88279 05704 94617 89655 74112 98111 94085 78059 18701 98661 53268 32044
c(20) =	-0.00030 07176 71200 56376 19066 02886 49784 35376 31145 96634 48783 28920 64831 64974 58934 43688
c(21) =	0.00020 04813 77049 05741 94009 82011 47684 33453 36157 87241 63377 88621 21677 06693 48612 50599
c(22) =	-0.00013 36554 23459 29399 54756 50331 27853 01242 01755 85915 24419 25690 77270 29904 26756 58796
c(23) =	0.00008 91040 84561 05664 04004 21412 26639 38782 39331 08834 30564 32635 48171 94400 32978 93219
c(24) =	-0.00005 94029 10632 31535 13012 25167 21857 78346 10450 64335 77096 25310 08342 49411 34066 18863
c(25) =	0.00003 96020 15464 87878 36361 72078 33491 09552 06150 62389 95320 37589 33484 14582 24681 89622
c(26) =	-0.00002 64013 73662 48702 57551 88087 54784 96532 13206 92752 77718 82657 34241 25067 91753 13501
c(27) =	0.00001 76009 27783 22951 64529 72681 33788 27811 77736 52187 06775 10448 67827 67777 76442 37939
c(28) =	-0.00001 17339 56658 93706 36684 37940 59902 36069 50655 16376 27388 49518 25769 32793 00087 62999
c(29) =	0.00000 78226 39694 04943 19435 33876 71525 53840 47916 30500 55934 03932 23113 18428 07044 40105
c(30) =	-0.00000 52150 93897 94887 37143 33551 93104 39418 36686 35542 06709 44825 84931 43113 60123 96866
c(31) =	0.00000 34767 29572 73591 15713 22171 61285 64384 05000 41716 85149 38869 57508 51864 30836 35189
c(32) =	-0.00000 23178 19838 13297 60470 39693 92435 15019 68400 22859 52331 45919 28968 78368 92847 71196
c(33) =	0.00000 15452 13274 61256 09181 90541 25142 92738 80392 18072 17360 28503 82135 41330 24529 89758
c(34) =	-0.00000 10301 42202 75135 60058 92014 61242 31274 52613 15377 41648 31494 34995 45801 34496 86703
c(35) =	0.00000 06867 61476 37145 43450 64507 45562 17745 85887 55205 77053 80494 04224 73002 52641 18131
c(36) =	-0.00000 04578 40987 39586 32763 38171 82045 54054 26493 96367 12796 71874 62121 69597 48888 36072
c(37) =	0.00000 03052 27326 18986 83103 68655 64826 08903 33865 47656 61817 11737 72017 02282 81470 17594
c(38) =	-0.00000 02034 84884 63029 65547 97896 18473 99005 24248 74575 58748 51090 04102 55916 66547 94125
c(39) =	0.00000 01356 56589 95501 82701 91952 37968 76252 71942 70730 71092 90739 28727 86109 37244 82109
c(40) =	-0.00000 00904 37726 71727 37863 08497 15425 60565 38098 48512 37762 17889 03701 01031 93282 74830
c(41) =	0.00000 00602 91817 84375 38465 01769 93898 55945 06013 35063 94966 85861 49314 14662 85586 25375
c(42) =	-0.00000 00401 94545 24206 44303 45524 37362 97737 00774 07221 33401 58542 14733 49092 13827 53339
c(43) =	0.00000 00267 96363 49986 77010 42160 90373 56985 36663 90711 69859 40713 98770 03761 39860 11234
c(44) =	-0.00000 00178 64242 33530 83666 59711 33835 57223 12310 02056 00685 14945 73265 37316 10901 25641
c(45) =	0.00000 00119 09494 89103 08709 13462 69578 41772 91879 71520 63867 90118 24265 02730 17928 86417
c(46) =	-0.00000 00079 39663 26101 73645 56822 06567 44089 70669 48459 04685 63050 59462 57210 35342 23790
c(47) =	0.00000 00052 93108 84081 02899 57511 08455 25307 28220 19044 74985 66930 35495 06481 64485 88413
c(48) =	-0.00000 00035 28739 22725 96787 41570 97129 66874 04484 52906 40263 72065 77619 56691 43907 93965
c(49) =	0.00000 00023 52492 81819 42466 69590 37576 20462 03941 44952 09392 91776 48070 49350 64170 91649
c(50) =	-0.00000 00015 68328 54547 12821 16644 41689 81494 39758 28853 73731 73620 29860 69147 44477 85036
c(51) =	0.00000 00010 45552 36365 09018 12577 64217 21101 19877 12791 34863 83922 62313 52187 57468 01825
c(52) =	-0.00000 00006 97034 90910 19533 68991 26839 53340 14203 06814 69181 21993 99273 23125 99544 06742

TABLE V

Values of the coefficients in the expansion of the inverted Gamma function for $s = 3/2$.

d(1) =	1.12837 91670 95512 57389 61589 03121 54517 16881 01258 65799 77136 88171 44342 12849 36882 98683
d(2) =	-0.04117 45264 45283 10145 02472 05115 70419 01750 06113 89637 71286 25112 74615 60795 59704 31681
d(3) =	-0.52665 44355 25544 47926 32079 72841 09288 56032 64837 55682 29889 27550 59489 95837 80514 92687
d(4) =	0.17510 20260 43934 56149 51226 18525 57115 01517 82399 43893 90470 02625 73625 82513 61031 60628
d(5) =	0.05096 68602 47706 07677 46983 49409 75961 84704 59543 20866 62027 58435 19671 90962 96174 66347
d(6) =	-0.04215 51693 68535 60099 31854 35843 46657 85781 06883 49080 02276 12384 49631 63043 85193 72522
d(7) =	0.00661 28978 26824 12727 65662 56030 54543 62835 01817 64006 03074 35963 45731 65790 77870 53574
d(8) =	0.00212 07314 42572 93833 60118 52790 99258 13126 70664 81192 91585 49409 76582 99680 35325 44544
d(9) =	-0.00111 07302 54594 89071 71194 98478 86160 64859 35345 62990 25123 08439 06982 69138 39347 47839
d(10) =	0.00015 23576 20767 47687 21655 65418 67277 37230 10050 44436 30714 55534 29774 16969 00130 22408
d(11) =	0.00002 53552 04923 81416 52782 52816 10103 66226 94458 49751 44008 40150 53067 66890 55699 18599
d(12) =	-0.00001 38968 05717 91375 60219 66503 95717 70842 77581 75530 38754 33480 20611 52146 09153 36819
d(13) =	0.00000 21562 03290 51417 24534 55616 23949 45218 76113 90476 78306 41166 47242 92559 96346 63859
d(14) =	0.00000 00579 42640 54052 67250 42262 33219 53071 59604 91641 74158 79757 64532 20495 75252 98682
d(15) =	-0.00000 00891 35511 18311 11605 40721 02332 51438 73328 32204 81665 68336 39829 28840 42062 90282
d(16) =	0.00000 00171 03469 41591 53737 49320 41168 64490 33182 09015 74846 64867 87406 65644 16245 23533
d(17) =	-0.00000 00009 31368 64452 41901 56847 57121 03992 04377 10204 26376 63538 14457 12126 51131 38061
d(18) =	-0.00000 00002 68047 41033 49662 55650 42604 12811 57723 47782 92912 33769 49097 97790 46589 61604
d(19) =	0.00000 00000 74589 32233 31632 60506 92751 48471 72632 51046 47969 74368 04065 52473 64610 46225
d(20) =	-0.00000 00000 08012 80706 14147 18370 91842 48691 23608 79618 12302 19768 62240 23604 19783 89465
d(21) =	-0.00000 00000 00083 82343 03345 18549 30489 93806 33902 12506 66315 30665 00298 02797 60315 32813
d(22) =	0.00000 00000 00169 46340 90432 05222 67740 94974 38014 13022 65545 63535 59019 78203 07906 97520
d(23) =	-0.00000 00000 00027 87575 67071 25752 08297 53147 96493 29340 82681 07805 36761 79478 59550 19168
d(24) =	0.00000 00000 00001 86703 94695 06530 54191 19188 40589 11440 53485 06186 66982 61684 91953 71864
d(25) =	0.00000 00000 00000 13049 49900 85879 86588 17799 98927 35780 40861 92882 68833 50321 38856 35928
d(26) =	-0.00000 00000 00000 04858 87414 41877 86529 61731 08402 98295 71065 90453 72022 35628 34674 10040
d(27) =	0.00000 00000 00000 00582 95426 92459 46783 18599 84244 21052 15780 29982 96525 21865 05927 44168
d(28) =	-0.00000 00000 00000 00025 92909 41799 37838 70477 84514 25919 18218 95695 57926 68398 65789 88532
d(29) =	-0.00000 00000 00000 00003 32675 40102 85788 59871 42882 20413 88729 54026 03708 80374 40064 46931
d(30) =	0.00000 00000 00000 00000 79449 61635 76810 53924 96881 65457 63873 70190 56982 80506 50196 09257
d(31) =	-0.00000 00000 00000 00000 07755 54328 84373 57293 90253 77073 16772 37834 35449 41906 17538 96607
d(32) =	0.00000 00000 00000 00000 00255 33736 29132 96957 91806 30480 15362 58531 88942 07410 97570 30235
d(33) =	0.00000 00000 00000 00000 00042 74520 16014 71733 79460 02920 72363 42310 60718 16496 48645 13088
d(34) =	-0.00000 00000 00000 00000 00008 26338 13746 68449 50568 79321 72514 28478 54547 49187 06390 08468
d(35) =	0.00000 00000 00000 00000 00000 71081 87657 25339 79736 55066 50482 39651 03124 66698 42696 82293
d(36) =	-0.00000 00000 00000 00000 00000 02074 94638 87704 29612 45013 10690 01248 14732 98561 32832 61884
d(37) =	-0.00000 00000 00000 00000 00000 00328 59544 06994 86048 30125 91386 18242 74687 86599 77266 97282
d(38) =	0.00000 00000 00000 00000 00000 00058 19433 90819 87478 73747 69479 02225 03415 36340 08314 23132
d(39) =	-0.00000 00000 00000 00000 00000 00004 70293 21304 49893 36039 90226 26453 86248 96238 86005 06248
d(40) =	0.00000 00000 00000 00000 00000 00000 14478 55651 58528 64715 71007 47334 16737 14946 90517 77690
d(41) =	0.00000 00000 00000 00000 00000 00000 01597 83150 54786 78867 34059 40543 53327 73835 18743 61494
d(42) =	-0.00000 00000 00000 00000 00000 00000 00287 82471 74761 34448 59106 01443 70749 76498 12799 76317
d(43) =	0.00000 00000 00000 00000 00000 00000 00023 00886 40061 20204 75117 23789 27644 25923 08048 86900
d(44) =	-0.00000 00000 00000 00000 00000 00000 00000 81524 69850 72319 69959 00845 41422 71516 81071 83108
d(45) =	-0.00000 00000 00000 00000 00000 00000 00000 04842 01452 13897 81552 19553 33904 52797 10271 67115
d(46) =	0.00000 00000 00000 00000 00000 00000 00000 01018 82590 41957 85489 64669 06836 98274 81330 98481
d(47) =	-0.00000 00000 00000 00000 00000 00000 00000 00084 12953 04954 03489 52788 55482 99738 59373 63814
d(48) =	0.00000 00000 00000 00000 00000 00000 00000 00003 48870 05836 57446 86098 51780 64378 28045 47036
d(49) =	0.00000 00000 00000 00000 00000 00000 00000 00000 07563 10899 94869 57784 54309 91541 18481 05670
d(50) =	-0.00000 00000 00000 00000 00000 00000 00000 00000 02589 53190 69537 33195 18160 84218 45828 88632
d(51) =	0.00000 00000 00000 00000 00000 00000 00000 00000 00230 70239 97849 78036 96572 53603 35271 82461
d(52) =	-0.00000 00000 00000 00000 00000 00000 00000 00000 00011 09196 43852 26435 46500 35654 57083 62269

TABLE VI

Values of the integral from a to b of the inverted Gamma function.

a	b	Value
-10	-9	2 60547.70793 20506 24303 22753 58692 01437 67668 48219 45086 41142 47077 28908
-9	-8	-27136.66927 95424 98804 74304 17037 11893 49938 94902 74756 63019 48137 25963
-8	-7	3156.81596 18461 08044 20868 33161 53950 15210 52315 75487 85943 49217 51207
-7	-6	-415.89693 20832 07590 12059 97940 79191 40021 67070 76448 45235 77151 79573
-6	-5	63.17186 75843 52479 94427 47404 14317 31537 90134 88805 14586 10927 46851
-5	-4	-11.33036 90334 53643 82938 34047 21839 05995 87457 39828 43339 67321 55875
-4	-3	2.48180 29455 21126 14719 41931 36672 91175 68743 69964 28280 37034 32296
-3	-2	-0.69865 99099 13205 54269 78540 82284 22044 99644 10869 97857 05792 28033
-2	-1	0.27577 30330 41742 06946 05785 09521 11579 20666 80306 05785 13265 54887
-1	0	-0.18372 07119 05075 62303 64064 07517 25872 20736 81031 96338 50692 60152
0	1	0.54123 57343 28670 53014 95373 28879 55201 38992 56206 64997 60973 58961
1	2	1.08514 26643 57470 08432 68666 42788 47468 07913 88749 51824 40151 55964
2	3	0.75184 97084 95638 58030 24034 16281 13389 20045 96553 91095 00942 05304
3	4	0.31183 45669 98537 56114 35447 32465 29676 07627 89771 02522 03021 96225
4	5	0.09201 87627 02450 04602 66032 93339 36466 74236 62051 68938 15300 98004
5	6	0.02104 69087 10163 38860 26792 04887 32958 68536 59161 40934 39519 65930
6	7	0.00392 75453 99260 25069 76775 05698 14214 55084 13884 54217 97648 31450
7	8	0.00061 87196 85282 88900 55916 45690 32071 10020 78927 14522 69898 31581
8	9	0.00008 43101 02376 96441 09833 24979 05141 63306 31927 10260 74513 96317
9	10	0.00001 01204 67243 01888 69534 55520 69096 16485 14307 66301 62907 10441
10	11	0.00000 10854 58619 45183 43671 84949 43201 90384 73051 34683 60422 04483
11	12	0.00000 01052 05646 89034 15165 91603 67371 87103 48315 10961 31278 45146
12	13	0.00000 00093 00396 95135 62725 18603 74292 22185 05312 38377 27314 61803
13	14	0.00000 00007 55703 23632 16225 78532 98253 65890 55909 97059 89052 88909
14	15	0.00000 00000 56809 61723 34546 11600 98577 05081 37985 38393 09207 00059
15	16	0.00000 00000 03973 20013 40782 13593 23653 18664 36230 77679 78222 57915
16	17	0.00000 00000 00259 78223 06816 40553 49907 80379 48734 80667 50933 99216
17	18	0.00000 00000 00015 94657 92540 62504 27622 85991 87165 31961 93750 66759
18	19	0.00000 00000 00000 92243 61105 15693 74263 85586 84367 05432 23009 50137
19	20	0.00000 00000 00000 05044 94534 03530 72798 94698 76182 18409 90637 30357
20	21	0.00000 00000 00000 00261 64749 32751 81510 99875 52328 28011 12536 85480
21	22	0.00000 00000 00000 00012 90255 88783 50632 68074 76806 74769 91081 78224
22	23	0.00000 00000 00000 00000 60643 28149 31565 65824 94574 49989 06280 73563
23	24	0.00000 00000 00000 00000 02722 63781 56648 95646 07305 97409 13802 58658
24	25	0.00000 00000 00000 00000 00116 99491 96378 39095 36558 86531 28603 94208
25	26	0.00000 00000 00000 00000 00004 82070 97057 38511 32772 92431 03110 96429
26	27	0.00000 00000 00000 00000 00000 19078 96094 17036 94107 69823 32783 72517
27	28	0.00000 00000 00000 00000 00000 00726 39787 21487 98659 98742 55453 34646
28	29	0.00000 00000 00000 00000 00000 00026 64382 56603 50770 93457 89274 90625
29	30	0.00000 00000 00000 00000 00000 00000 94276 17992 14809 55078 33636 12144
30	31	0.00000 00000 00000 00000 00000 00000 03222 04589 77308 87021 30986 47037
31	32	0.00000 00000 00000 00000 00000 00000 00106 48553 43188 38011 72724 70892
32	33	0.00000 00000 00000 00000 00000 00000 00003 40683 24048 39273 38385 93189
33	34	0.00000 00000 00000 00000 00000 00000 00000 10562 20763 81559 13715 32268
34	35	0.00000 00000 00000 00000 00000 00000 00000 00317 62744 07275 99815 96835
35	36	0.00000 00000 00000 00000 00000 00000 00000 00009 27324 62655 67454 45996
36	37	0.00000 00000 00000 00000 00000 00000 00000 00000 26306 59850 82825 95610
37	38	0.00000 00000 00000 00000 00000 00000 00000 00000 00725 71289 69607 28736
38	39	0.00000 00000 00000 00000 00000 00000 00000 00000 00019 48325 28602 32404
39	40	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 50940 87870 57866
40	41	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 01298 00257 13773
41	42	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00032 25297 72329
42	43	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 78201 74373
43	44	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 01851 27060
44	45	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00042 81272
45	46	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 96774
46	47	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 02139
47	48	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00046
48	49	0.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00001
	F	2.80777 02420 28519 36522 15011 86557 77293 23080 85920 93019 82912 20055

TABLE VII

Values of Gamma(p/q).

p	q	Value
1	10	9.51350 76986 68731 83629 24871 77265 40219 25505 78626 08837 73430 50000 77043 42653 83322 82101
1	9	8.52268 81392 19475 95051 43922 14439 55975 47588 31469 32202 08985 32701 61790 58870 16992 95138
1	8	7.53394 15987 97611 90469 92298 41215 13362 46104 19588 14907 59409 83127 89777 66636 57198 90641
1	7	6.54806 29402 47824 43771 40933 49428 99626 26211 35187 38413 51489 40168 81914 85762 04738 23914
1	6	5.56631 60017 80235 20425 00968 95207 72611 13987 99114 87285 34616 16744 62632 29075 02817 80231
1	5	4.59084 37119 98803 05320 47582 75929 15200 34341 09998 29340 30177 88853 13623 00392 73106 44500
2	9	4.10657 95667 16061 19664 22907 88918 36054 03210 52485 15292 42469 64982 38258 66838 99761 46476
1	4	3.62560 99082 21908 31193 06851 55867 67200 29951 67682 88006 54674 33377 99956 99192 43538 72912
2	7	3.14911 51177 59936 59097 01136 64680 76889 22297 78611 76625 26847 90761 50003 94279 84532 96946
3	10	2.99156 89876 87590 62831 25165 15904 91779 11128 06024 92171 51127 44119 65095 63887 67876 32022
1	3	2.67893 85347 07747 63365 56929 40974 67764 41286 89377 95730 11009 50428 32759 04176 10167 74382
3	8	2.37043 61844 16600 90864 64735 04176 65250 98874 00803 35892 49877 75126 93467 31615 31358 00179
2	5	2.21815 95437 57688 22305 90540 21907 67945 07705 66501 77146 95822 41977 75264 61851 68123 00474
3	7	2.06751 17265 60229 35302 46124 06308 82694 35592 14211 49238 75280 50717 59023 46033 90293 97673
4	9	1.99289 35227 56922 75771 82035 62637 98751 71413 60387 65003 04062 92209 96814 24773 53781 19038
1	2	1.77245 38509 05516 02729 81674 83341 14518 27975 49456 12238 71282 13807 78985 29112 84591 03218
5	9	1.60071 61184 13983 28967 76129 26406 83369 71616 21038 83996 53728 30023 76424 22664 60581 80870
4	7	1.55858 10329 02475 00827 50092 91245 97392 25208 50472 09453 86922 66736 62932 89725 58387 84230
3	5	1.48919 22488 12817 10239 43333 88321 34228 13205 99038 75992 47353 38679 56404 50801 63121 93494
5	8	1.43451 88480 90556 77563 60197 39456 42313 66322 07772 20666 73307 70679 85809 50941 97302 09691
2	3	1.35411 79394 26400 41694 52880 28154 51378 55193 27266 05679 36983 94022 46796 37829 65401 74254
7	10	1.29805 53326 47557 78568 11711 79152 81161 77841 41170 55394 62479 21645 38825 41681 50818 97580
5	7	1.27599 26754 93444 05848 53056 07789 87494 84545 88992 91105 19162 28146 37620 71014 76123 92985
3	4	1.22541 67024 65177 64512 90983 03362 89052 68512 39248 10807 06112 30118 93828 98228 88426 79836
7	9	1.19015 11869 12872 71460 38590 53883 03526 48713 81437 77011 42320 24031 09342 67621 33617 86587
4	5	1.16422 97137 25303 37363 63209 38268 45869 31419 61768 89118 77529 84894 46786 18354 66078 95374
5	6	1.12878 70299 08125 96126 09010 90258 84201 33267 87441 66475 54517 52083 51433 37705 10987 50399
6	7	1.10576 70723 29567 32661 98492 94247 33752 92315 46976 82003 88489 45380 02358 64184 93347 92056
7	8	1.08965 23574 22896 95125 23767 55102 89297 11478 70067 76756 51205 13704 04325 36264 17465 87950
8	9	1.07775 88331 33495 79725 70063 33077 01632 92059 53740 72476 67863 58975 39844 05959 74420 55203
9	10	1.06862 87021 19319 35489 73053 35694 48077 81698 38785 06097 31790 49370 68398 15721 77025 44757
1	1	1.00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000 00000

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